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EXTRACTING THE MEDIAL AXIS FROM THE VORONOI DIAGRAM OF BOUNDARY SEGMENTS: AN ALTERNATIVE METHOD FOR CLOSED BOUNDARY DETECTION

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ABSTRACT

An algorithm to recover closed boundaries from disconnected boundary segments is presented. There is a close relation between the medial axis transform and the Voronoi diagram. Here we introduce a geometric labeling scheme for the Voronoi diagram of boundary segments, and recover the medial axes of closed boundaries by using the labeled Voronoi diagram. Although all examples given in this paper are pictures of straight line segments in the two-dimensional Euclidean plane, the basic idea is immediately applicable to digital pictures with curved segments.

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1. Introduction

The problem discussed in this paper is how to recover closed boundaries from a set of spatially separated boundary segments.* A commonly used method for this problem is curve linking. Although we can incorporate various geometric and sometimes gray level properties to select segments for the linking, these features are in essence very local. Since the closure of a boundary is a global property, some global criterion to re-evaluate local linking is required. As a result, an algorithm for this problem usually becomes complicated, and it is inevitable to introduce many heuristics (heuristic evaluation functions), which obscures the performance of the algorithm.

The algorithm presented in this paper does not use a curve linking scheme but recovers the medial axes of closed boundaries from the Voronoi diagram of the given segments. Fig. 1 shows our basic approach to closed boundary detection. The Voronoi diagram may be less local than the linking because a segment can interact with others at large distances through the Voronoi diagram. As will be discussed below, the Voronoi diagram represents various geometric properties such as the shapes of segments and geometric relations among them, whereas geometric properties of their endpoints are the major characteristics used in a curve linking algorithm. In this sense, using the Voronoi diagram, we will be able to incorporate a richer set of geometric properties for closed boundary detection. (Of course, since

*Generally, the input boundary segments form multiple closed boundaries.

information about shapes of the closed boundaries is missing, some heuristics are inevitable to recover them.)

The Voronoi diagram was originally defined for a point set in the plane (or in multi-dimensional space) to partition the space into disjoint subspaces. Each subspace is defined as the set of locations closest to a certain data point and can be considered as the neighborhood of that point. As is well known, the Voronoi diagram of a two-dimensional point set consists of convex polygons whose boundaries are defined by parts of the perpendicular bisectors between two points. (At marginal locations, the polygons become open.) We will call such polygons (subspaces) Voronoi regions and their edges Voronoi edges. Voronoi vertices are defined as locations where more than three Voronoi regions meet.

Recently, an O(N log N) algorithm to compute the Voronoi diagram of N points has been developed [1], and fast algorithms to construct the Voronoi diagram of a set of line segments (and circles) have also been proposed [2] [3]. In digital picture processing, algorithms for calculating the digital Voronoi diagram of a set of connected components have been proposed [4] [5]. Several papers have discussed applications of the Voronoi diagram to clustering and shape analysis of dot patterns [6] [7] [8].

Informally, the process of generating the Voronoi diagram of a set of objects can be described as follows. Imagine that the space is filled with some flammable material and a fire is started at each object. (If an object is not a point, the fire is started at each point on the object's boundary.) The fire spreads in all directions with equal speed. Then the locations where fire fronts from different objects meet form the Voronoi diagram. (Each fire front has a unique label corresponding to its originating object.)

This definition of the Voronoi diagram is exactly the same as the "grassfire" definition of the medial axis transform [4] [9]. The differences between them are:

(a) The Voronoi diagram is defined for a set of disjoint objects, while the medial axis transform is defined for a closed boundary.

(b) The medial axis contains the distance from each point on it to the nearest boundary.

In fact, Lee [10] proposed a fast algorithm to compute the medial axis of a polygon by using the Voronoi diagram. Fischler and Barrett [4] gave different labels to parts of a closed boundary and applied "Voronoi expansion" to generate a smooth medial axis of the boundary.

Our basic idea for recovering closed boundaries from a set of disjoint segments is as follows (Fig. 1). For a closed boundary, the medial axis is entirely embedded in the Voronoi diagram [10]. If the disjoint segments given as input data are points of closed boundaries,* their Voronoi diagram may contain information about the medial axes of the original closed boundaries. If we can extract the medial axes as subsets of the Voronoi diagram, we can easily recover the closed boundaries by using the "inverse"

*In this paper, we assume no noise segments are included. This assumption may not be unreasonable if we use a very high threshold to extract segments from the given picture.

medial axis transform. Of course, since boundaries are destroyed by gaps, medial axes extracted from the Voronoi diagram are approximations of the original ones, and some ambiguities may appear during the extraction process.

In what follows, we will explain the algorithm for extracting medial axes from the Voronoi diagram of a set of straight line segments in the two-dimensional Euclidean plane. The extension to curved segments will be considered in the discussion.

2. Geometric labeling of Voronoi edges

Suppose we are given a pair of line segments. In general, their Voronoi diagram is as shown in Fig. 2, where the space is divided into two half spaces (Voronoi regions) by a continuous curve (Voronoi edge). (Note that all figures in this paper are only for illustrative purposes; they were not obtained from real data.) The Voronoi edge is the locus of points at the same distance from the line segments, and is composed of five segments. According to the notation used in [2], these segments are B(a,c), B((a,b),c), B((a,b),(c,d)), B((a,b),d), and B(b,d). Here a,b,c, and d denote the endpoints of the line segments, (a,b) and (c,d) open line segments, and $B(e_i,e_j)$ is the locus of points equidistant from e_i and e_j. B(endpoint,endpoint) is the perpendicular bisector of the line segment connecting the two endpoints, B(open line segment, open line segment) the angular bisector of the angle defined by the two line segments, and B(endpoint, open line segment) the parabola whose focus is the endpoint. Lee [2] discussed several geometric properties of the Voronoi diagram of a set of line segments.

As is obvious from this example, the Voronoi diagram represents various geometric properties of the pairs of line segments. We will be able to use these properties for shape analysis. But, since each Voronoi edge is composed of several segments representing different geometric properties, we cannot regard it as a unit for shape analysis. Our fundamental idea in this paper is to partition each Voronoi edge into small segments by using the geometric labels (B(a,c), B(c,(a,b)), etc.) and to consider those segments as elements for processing. We will use three labels for segments, and call them P-P (point to point), P-L (point to line), and L-L (line to line) segments instead of B(a,c), B(c,(a,b)) and so on.

As shown in Fig. 2, a Voronoi edge changes its label when it crosses over one of the straight lines which are perpendicular to the line segments at their endpoints. These straight lines are loci of points equidistant from the endpoints and the (open) line segments. The rectangular regions bounded by two straight lines attached at two endpoints of a line segment represent "neighborhoods" of the endpoints and the line segment. A Voronoi edge is given either a "point" or "line" label from a line segment depending on which rectangular region it is located in. (The other label is given from the other line segment defining the Voronoi edge.)

The situation illustrated in Fig. 2 is just one example of the geometric relations between two line segments. Fig. 3 shows other examples, where the labeling changes according to the geometric relation between the segments. We will be able to extract various geometric properties by using this labeling scheme, such as symmetry, collinearity, parallelism, and so on.

An important property of this labeled Voronoi diagram is [Theorem 1]. Let L_1, L_2, L_3, and L_4 denote four half straight lines which are perpendicular to a line segment ℓ at its endpoints (Fig. 4). The border of the Voronoi region of ℓ intersects at most once with each of L_1-L_4.

Proof. Suppose the border of the Voronoi region intersects twice with L_1, and let P_1 and P_2 denote the intersections and e an endpoint of ℓ as shown in Fig. 4. Draw circles, C_1 and C_2, centered at p_1 and p_2 with radi of $\overline{P_1e}$ and $\overline{P_2e}$, respectively, where $\overline{P_1e}$ and $\overline{P_2e}$ denote distances between two points. Then the circle C_1 entirely includes C_2 except for e. This contradicts the definition of the Voronoi diagram; in order for P_2 to be on the Voronoi diagram, there must be at least one point P on C_2 which belongs to another line segment. Then, since $\overline{P_1P}<\overline{P_1e}$, P_1 cannot be a point on the border of the Voronoi region of ℓ. ($\overline{P_1e}$ is the shortest distance between P_1 and ℓ, and P_1 is nearer to P than ℓ.)

In this paper we do not describe an algorithm to realize the geometric labeling and assume that the Voronoi diagram is labeled and that each point on the diagram contains the distance to the nearest line segment.

3. Basic considerations for the algorithm

3.1 Case 1: Two line segments

Suppose the two line segments shown in Fig. 2 are parts of a closed boundary. What is a reasonable estimate of the medial axis of the boundary? It seems to be natural to consider an L-L segment (i.e., B((a,b),(c,d))) of the Voronoi edge as a part of the medial axis, because it is defined by the parts of the boundary in exactly the same way as the medial axis transform. On the other hand, P-P and P-L segments may be considered as side-effects of breaking the continuous boundary.

Fig. 5 shows the result of expanding the L-L segment in Fig. 2 by using distances recorded on the points of the segment (the "inverse" medial axis transform). The expanded region shares its boundary with the line segments and will intersect with no line segment even if there are many other line segments. (This is a fundamental property of the Voronoi diagram, and the proof is trivial.)

As shown in Fig. 5, some parts of the line segments are not included in the boundary of the expanded region. This is because P-L segments were excluded in the expansion. If we include P-L segments as well as L-L segments in the medial axis, the expanded region boundary will include the entire line segment as its parts. But, if there are other line segments, the region may intersect with some of them. Therefore, in order to determine whether or not P-L segments are to be included in the medial axis, we have to consider geometric relations to other line segments.

Based on these observations, the following conjecture may be reasonable:

[Conjecture] Parts of the medial axis of a closed boundary are preserved as L-L segments in the Voronoi diagram even if the boundary is broken into disjoint segments.

This conjecture holds in cases where the erased (missing) parts of a boundary are small and simple. Our algorithm uses this conjecture as its basis. (Section 5 includes a discussion about this conjecture.)

3.2 Case 2: Convex boundary

Here we assume the line segments are parts of a convex polygon. In this case, all L-L segments in the Voronoi diagram are entirely included in the original polygon, and most of the line segments form L-L segments in the Voronoi diagram.

Another interpretation of an L-L segment is that the region expanded from it specifies a convex area defined by two line segments forming the L-L segment (see Figs. 2 and 3). Therefore, in the case of a convex boundary, the regions expanded from L-L segments usually overlap with each other. If the gaps on the boundary are small, all such regions are merged into one connected region. This merged region is a "conservative" estimate for the original closed boundary, where "conservative" means that the entire region is included in the interior of the boundary.

Since the L-L segments in the Voronoi diagram are usually not connected, in order to extract (estimate) the medial axis from the Voronoi diagram, it is necessary to connect them into a connected graph by using other segments in the Voronoi diagram. In the case of a convex boundary, the connection of L-L segments can be easily performed by tracking Voronoi edges from one L-L segment to another until all L-L segments are connected. Then the medial axis is extracted as a connected graph on the Voronoi diagram which includes all L-L segments. However, if a concave boundary and/or multiple boundaries are included in a picture, the simple tracking algorithm is not sufficient.

3.3 Case 3: Concave polygon

Fig. 6 shows an example of the labeled Voronoi diagram of a concave boundary. In this case, L-L segments are formed on both sides of the boundary. This is because an L-L segment is just a local geometric property showing a convex area defined by two line segments. Some global process is necessary to determine to which side of the boundary each convex area belongs.

In order to fill gaps between line segments, it might seem reasonable to link the endpoints of two line segments forming an L-L segment. But as shown in Fig. 6, L-L segments are generated not only by line segments "consecutive" along an original boundary but also by a pair of facing line segments on the opposite sides of the boundary. Therefore, such a linking process would unnaturally divide the boundary.

In short, L-L segments in the Voronoi diagram are important keys for recovering the medial axis, but some global process is necessary for checking consistency among them (that is, whether they specify the inside or the outside of a boundary).

4. Algorithm

We will now describe our algorithm to extract medial axes from the Voronoi diagram of a set of line segments. The algorithm usually extracts multiple connected subgraphs from the Voronoi diagram and regards them as the medial axes of closed boundaries.

We assume that the Voronoi edges are labeled and segmented into L-L, P-L, and P-P segments, and use the following notation.

$L = \{\ell_1, \ell_2, \ldots, \ell_n\}$: set of line segments

P_{i1}, P_{i2} : endpoints of a line segment ℓ_i

$LL = \{LL_{ijh} : h=1 \text{ or } 2\}$: set of L-L segments, where LL_{ijh} denotes an L-L segment between ℓ_i and ℓ_j*,**

$PL = \{PL_{ihj} : h=1 \text{ or } 2\}$: set of P-L segments, where PL_{ihj} denotes the P-L segment between P_{ih} and ℓ_j*

$PP = PP_{ihjk} : h,k=1 \text{ or } 2$: set of P-P segments, where PP_{ihjk} denotes the P-P segment between P_{ih} and P_{jk}*.

Since the ordinary algorithms for constructing the Voronoi diagram do not use these geometric labels, we have to modify them to obtain the labeled Voronoi diagram.

* From Theorem 1, these symbols are unique.

** Note that at most two disjoint L-L segments can be formed between a pair of line segments (see Fig. 3(b)).

4.1 STEP 1: Erasing redundant P-P segments

There are many obviously redundant P-P segments which extend to infinity. We can remove these segments from the Voronoi diagram. Note that neither L-L nor P-L segments extend to infinity, because the rectangular regions where they are included are "bounded" (see Fig. 3). (In the case of a bounded picture, we can remove these P-P segments touching picture edges, assuming that there is enough open space around a set of line segments.)

If a P-P segment PP_{ihjk} extends to infinity, we can connect P_{ih} and P_{jk} by a straight line segment without intersecting any other line segments. By connecting all pairs of endpoints that form P-P segments extending to infinity, a continuous convex hull for the set of line segments is produced. This property is immediate from Theorem 2 in [2]. The addition of these new line segments to an original set of line segments facilitates later processing:

(i) The addition should not cause any artifacts in the method of medial axis extraction (closed boundary detection).

(ii) Many new L-L segments are produced by connecting P_{ih} and P_{jk}, which gives more clues for medial axis extraction.

(iii) By adding new line segments, open boundaries like Fig. 6 become closed, so that we can process such open boundaries as well as closed boundaries by our algorithm.

Therefore, we include these line segments into the data set, and compute the Voronoi diagram of the new data set.

4.2 STEP 2: Finding relations among L-L segments

The next step is to extract all L-L segments and to find geometric relations among them. We define two relations among L-L segments:consistent and contradicting. The consistent relation means that L-L segments with this relation should be connected into one medial axis, and the contradicting relation means that L-L segments with this relation should not be connected into one medial axis.

These two relations are established for a pair of L-L segments sharing a common line segment, respectively, that is, LL_{ijh} and LL_{pqk} such that i=p, i=q, j=p, or j=q. If two regions expanded from LL_{ijh} and LL_{pqk} dominate the different sides of the line segment, we say LL_{ijh} and LL_{pqk} are mutually contradicting because if they are included in a medial axis, the line segment becomes a "crack" in the region generated from the medial axis. If the regions expanded from LL_{ijh} and LL_{pqk} are located on the same side of the line segment, in the case of consistent L-L segments, then they should be connected into a medial axis. There are several ways to discriminate the above two situations:

(i) Use the coordinates of a given point on an L-L segment and decide on which side of a line segment it is located.

(ii) Give two different labels to a line segment which represent the two sides of the line segment, respectively, and construct the labeled Voronoi diagram.

(iii) Use the following Theorem, and decide on a relation between L-L segments by tracking the Voronoi diagram.

[Theorem 2]. Suppose two L-L segments share a line segment ℓ_I. If they are consistent, there must be a path on the Voronoi diagram connecting them which consists of LL_{Iqh} or PL_{jkI}. If there is no such path, then they are contradicting.

Proof. See Fig. 7. Consistent L-L segments sharing ℓ_I must be located in one of the two rectangular spaces defined by ℓ_I and two perpendicular straight lines at its endpoints. Since the border of the Voronoi region of ℓ_I is continuous, there must be Voronoi edges connecting the L-L segments in that rectangular space. From Theorem 1, such Voronoi edges must be labeled with LL_{Iqh} or PL_{jkI} because they are nearer to ℓ_I than to its endpoints. The proof for contradicting L-L segments follows trivially from the above proof.

Note that in the special case as shown in Fig. 3(h), a pair of L-L segments satisfy the consistent relation as well as the contradicting relation, depending on from which of the two line segments we consider the relations. In this case, we use the contradicting relation and do not regard the two L-L segments as consistent. Discussion about more complicated situations will be given in Section 5.

Since method (iii) gives a path on the Voronoi diagram to be included in the medial axis as well as discriminating between the two situations, we use this method for the algorithm. This path checking process can be easily realized by using a graph, where each node represents a labeled segment and links between nodes represent mutual connections between labeled segments on the Voronoi diagram.

Using the consistent relation among L-L segments, we can partition the set of L-L segments into disjoint subsets (LL^i) : $LL=\cup LL^i, LL^i \cap LL^j=\emptyset (i \neq j)$. To connect those L-L segments in the same subset by Voronoi edges is the purpose of the next step. The contradicting relations will be used as constraints for extending consistent L-L segments to medial axes.

Note that if our basic conjecture holds, no pair of L-L segments in the same subset have the contradicting relation. The following algorithm assumes this condition. See Section 5 for the processing in case this assumption is invalid.

4.3 STEP 3: Extending L-L segments

Since L-L segments are small parts of medial axes and are usually disconnected, we have to extend them to form connected medial axes. As explained in the previous section, L-L segments included in the same subset (sharing a line segment ℓ_I) can be connected via PL_{jkI} segments. By including such PL_{jkI} in a subset, each subset comes to correspond to a connected subgraph of the Voronoi diagram. Note that a subgraph can have a loop when an original closed boundary has another boundary in its interior.

Using the same idea, each L-L segment LL_{IJP} can be extended to the adjacent PL_{ihI} and PL_{jkJ}. But in order to prevent contradicting subgraphs from being connected into one subgraph, we must not add a series of P-L segments which connect contradicting L-L segments. (See the remark following the proof of Theorem 2.) If some subgraphs become connected by this extension, we merge them into one subgraph, and define contradicting relations among the subgraphs.

4.4 STEP 4: Augmenting the set of subgraphs

By STEP 1-STEP 3 a set of connected subgraphs of the labeled Voronoi diagram consisting of L-L and P-L segments have been extracted. They define convex areas in the space bounded by the line segments (Fig. 8). We can use these subgraphs to segment complex polygons into simple convex polygons.

All L-L segments are included in the subgraphs. In most cases, all P-L segments are also merged into them, because P-L segments are usually adjacent to L-L segments. In special cases such as shown in Fig. 9, however, some P-L segments are isolated and not included in any subgraph. Since these isolated P-L segments can also be considered as representing convex areas, we process them in the same way as L-L segments: merge adjacent ones into one subgraph and establish contradicting relations to others (other connected P-L segments and the set of connected subgraphs extended from L-L segments). Here again, we need a special process in the situation as shown in Fig. 3(h): regard a P-L segment connecting a pair of contradicting L-L segments as an isolated subgraph, and do not connect it to the adjacent L-L segments. Then, a set of connected subgraphs on the Voronoi diagram including all L-L and P-L segments is obtained. Let $G=\{G_i\}$ denote this set. Usually, the subgraphs are disjoint on the labeled Voronoi diagram, and G_i and G_j $(i \neq j)$ have a contradicting relation if they include L-L or P-L segments defined by the same (open) line segment (not endpoint).

4.5 STEP 5: Removing P-P segments on a path connecting contradicting subgraphs

Since the labeled Voronoi diagram is connected even after STEP 1, there are Voronoi edges (usually P-P segments) connecting contradicting subgraphs. Thus, if a P-P segment directly connects two contradicting subgraphs, we regard it as redundant (side-effect of a gap) and remove it. By this process, some subgraphs are isolated from the others (Fig. 10). Since all contradicting subgraphs are not directly connected by a single P-P segment, some of them are still connected.

In order to disconnect contradicting subgraphs, we first find a path connecting two contradicting subgraphs G_i and G_j. It is easy to find the path to be disconnected by following the border of the Voronoi region of the line segment ℓ_I so that LL_{IQh} or PL_{jkI} are included in both of G_i and G_j (Fig. 11). It is obvious that this path consists of P-P segments and/or isolated P-L segments (no LL-segment is included).

If the path consists of P-P segments alone, we cannot use the contradicting relation to determine which one is to be removed. Fig. 12 illustrates a typical example of this situation. One heuristic to resolve this conflict is as follows:

(i) Expand G_i and G_j to regions R_i and R_j using the distances on them.

(ii) Let P_k ($k=1\sim m$) denote the endpoints of P-P segments on the path. P_1 and P_m are endpoints of G_i and G_j respectively. We assume the numbering is done from G_i to G_j. Find a k^* that satisfies the following conditions:

$$C_{k*} \cap R_i > C_{k*} \cap R_j \quad \text{and} \quad C_{k*+1} \cap R_i < C_{k*+1} \cap R_j$$

Here C_k denotes a circular region expanded from P_k. Since C_1 and C_k are entirely included in R_i and R_j respectively, there must be a k* satisfying the above inequalities $(2 \le k^* \le m-1)$

(iii) Remove a P-P segment between P_{k*} and P_{k*+1}.

Although process (ii) is asymmetric, this may not be crucial.

If the path includes P-L segments, local processing along the border of the Voronoi region of ℓ_I alone is not sufficient because the P-L segments have contradicting relations to other subgraphs. Our algorithm uses the following heuristics. Here P_k (k=1,...,m) denote endpoints of labeled (P-P or P-L) segments on the path P_1 and P_m denote the endpoints of two contradicting subgraphs G_i and G_j respectively (Fig. 10).

(i) Starting from P_1 (P_m),

if P_1P_2 (P_mP_{m-1}) is a P-P segment, then merge it to G_i (G_j), where P_1P_2 denotes the labeled segment between P_1 and P_2.

if P_1P_2 (P_mP_{m-1}) is a P-L segment, then

(i-1) if the P-L segment belongs to a subgraph contradicting G_i (G_j), then stop.

(i-2) if the P-L segment belongs to a path connecting G_j (G_i) and $\overline{G}_i$ ($\overline{G}_j$), then stop, where $\overline{G}_i$ ($\overline{G}_j$) denotes a contradicting subgraph of G_i (G_j) (see Fig. 13).

(i-3) otherwise, merge the P-L segment to G_i (G_j).

(ii) Repeat (i) until the merging process stops or comes up with P_m (P_1).

(The above process is performed from P_1 and P_m respectively.)

(iii) Let P_i and P_j denote endpoints of the extended subgraphs on the path $(1 \le i,\ j \le m)$.

(iii-1) If $i<j$, remove $P_{i-1}P_i$ and P_jP_{j+1}. These segments must be P-P segments; all adjacent L-L and P-L segments were merged into connected subgraphs by STEP 3 and 4 (except in the special case shown in Fig. 3(h)), and contradicting relations are defined among the subgraphs. Therefore, the segment merged just before the above process steps must be a P-P segment. (For the same reason, $i \neq j$.)

(iii-2) If $i>j$, the segments between P_j and P_i are included in both extended subgraphs (Fig. 3(h) is included in this category). This situation is very unusual, and there are several possible solutions.

(a) Regard the segments between P_j and P_i as an independent subgraph: if P_jP_{j+1} and $P_{i=1}P_i$ are P-P segments, then remove them. Otherwise put a special mark at P_j and P_i showing the endpoints of the subgraphs.

(b) Merge the shared segments to either G_i or G_j by using the same process as used before (expand skeletons and compare area sizes of overlapping regions).

(c) Remove the shared segments.

Since it is usually easier to merge divided objects than to split a merged object, (a) and (c) may be better than (b).

By applying the above process to each path connecting contradicting subgraphs, the entire Voronoi diagram is divided into disjoint subgraphs. In other words, the subgraphs extracted in STEP 3 and 4 are usually merged if they are not contradicting. We can then regard each subgraph as a medial axis.

4.6 STEP 6: Closed boundary generation

In order to complete the boundaries, we use the following post-processing.

(i) Expand each medial axis into a region using distances on the axis.

(ii) Track the boundary of the expanded region and order the line segments and their endpoints touching the boundary according to the order of the boundary tracking. (If a line segment is one included at STEP 1, remove it from the ordered set of line segments).

(iii) Apply a gap filling algorithm like [11] to each gap between successive endpoints of line segments.

Since the shape of the medial axis is deformed in neighborhoods of gaps, it is difficult to obtain smooth gap filling from the medial axis.

5. Discussion

In principle, our algorithm first extracts convex areas bounded by line segments and merges them unless they are contradicting. The detection of convex areas is based on exact geometric relations among line segments, whereas the merging process incorporates several heuristics. In this sense, our method is still too local to obtain reasonable results in complex situations. If a model of object shape is known, it may be better to control the merging process by using the model.

Another interpretation of the basic conjecture described in Section 3.1 is that a convex area defined by an L-L segment must belong properly to either the interior or exterior region of a closed boundary. This assumption usually holds if the gaps at the corners of a boundary are small. However, if an original closed boundary includes another closed boundary in its interior, (a region with a hole) and the gaps at the corners are very large, some areas defined by L-L segments will be included in both interior and exterior regions. In other words, "false" L-L segments are generated (Fig. 14(a)). As shown in Fig. 14(b), false L-L segments cause conflict between the consistent and contradicting relations. (Fig. 3(h) was the simplest example.) In other words, L-L segments in a consistent subset (defined in STEP 2) will have the contradicting relation. Since the contradicting relation is stronger than the consistent relation, we cannot use the consistent relations to merge the L-L segments in this case. If we enforced the consistent relation in the case of Fig. 14, we would obtain an ambiguity in the merging process: with which of A and C should B and D be merged? Therefore we need to add another step in the algorithm:

STEP 2.5 If any pair of L-L segments in a consistent subset satisfies the contradicting relation, cancel the subset and change each consistent relation in the subset into the contradicting relation. Note that mutually connected L-L segments are regarded as a consistent subset.

Cancellation of the subset is not enough, because B and D would be merged with A or C (by STEP 3) depending on the order of the merging process. STEP 5 in the algorithm determines the merging of P-L segments among these subgraphs. Since the algorithm does not know the "correct" global boundary to be detected, it is more reasonable to produce a set of partial results than to overmerge them without a clear criterion. By STEP 2.5, our algorithm can detect strange situations where its basic assumption is invalid. Therefore, it is possible to use this information to correct the input data by activating picture processing programs. For example, since subgraphs A, B, C, and D in Fig. 14 cause a conflicting situation, we can examine the related line segments ℓ_2, ℓ_5, ℓ_6, and ℓ_7 using picture processing programs. We should then be able to extend the short line segment ℓ_6 or to remove it as noise.

Note that our algorithm works well even for data with large gaps unless false L-L segments are generated. (Fig. 15).

It may be interesting to compare our method to curve linking. As explained in Section 2, our method relies on the L-L segment defined by a pair of line segments. Therefore, if two line segments do not form an L-L segment as shown in Figs. 3(b)(d), no useful geometric relation is defined between them. In other words, the convex areas defined by line segments are key characteristics in our closed boundary detection process. On the other hand, Figs. 3(b)(d) represent good clues for curve linking. The evaluation function for local curve linking gives high confidence in these cases, while it gives relatively low values to the other cases shown in Fig. 3. That is, its key feature is smooth continuation between segments. In this sense, our method and curve linking are mutually complementary. A combination of these methods should give more reasonable performance: first apply curve linking to obtain long curve segments, then use our method to detect closed boundaries. The long curve segments greatly facilitate our algorithm.

Although the algorithm described in this paper is for a set of line segments, it is also applicable to curve segments. In this case, we need some preprocessing; if a curve segment includes sharp corners, we divide it into smaller segments at the corners and give a different label to each segment.

As is well known, the medial axis transform has an intrinsic problem: the structure of the medial axis is sensitive to noise. The Voronoi diagram is also subject to the same problem, and sometimes its structure is greatly changed by adding noise (noisy segments) and changing the locations of data segments. One way to avoid noisy segments is to use a very strict threshold for extracting data segments. Another method is to extract long segments by curve linking and remove shorter ones. As mentioned above, it is also possible to use feedback analysis to examine data segments which cause a conflicting situation between the consistent and contradicting relations, because additional noisy segments usually lead to such a conflict. Although our method is sensitive to the locations of the endpoints of the line segments, detecting long segments by linking can reduce this sensitivity.

6. Conclusion

We have proposed an algorithm to extract the medial axis from the Voronoi diagram of a set of line segments. The basic idea was to give geometric labels to parts of the Voronoi edges so that precise geometric relations between line segments may be encoded in the Voronoi diagram. This idea is also applicable to the Voronoi diagram of a set of points and blobs to detect its "perceptual" boundaries. As mentioned in Section 4, our algorithm can be immediately used to partition a complex region into a set of simple convex parts. Algorithms to compute the labeled Voronoi diagram and to extract medial axes in a digital picture are currently under implementation.

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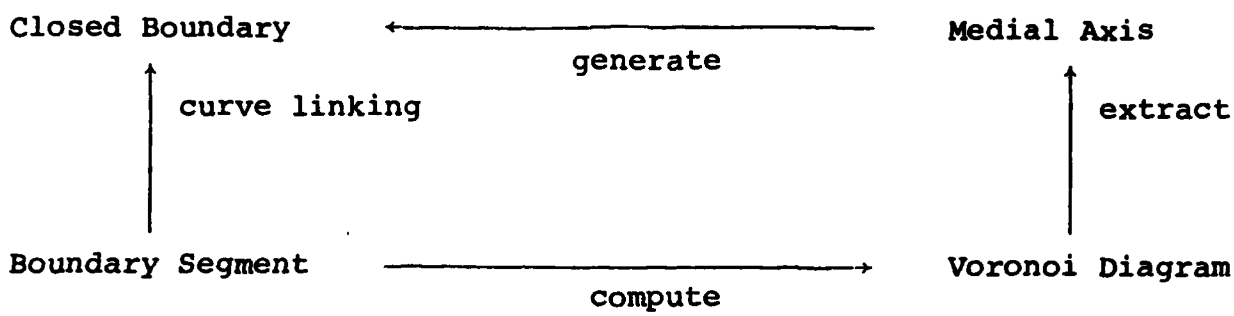


Figure 1. Basic scheme for closed boundary detection.

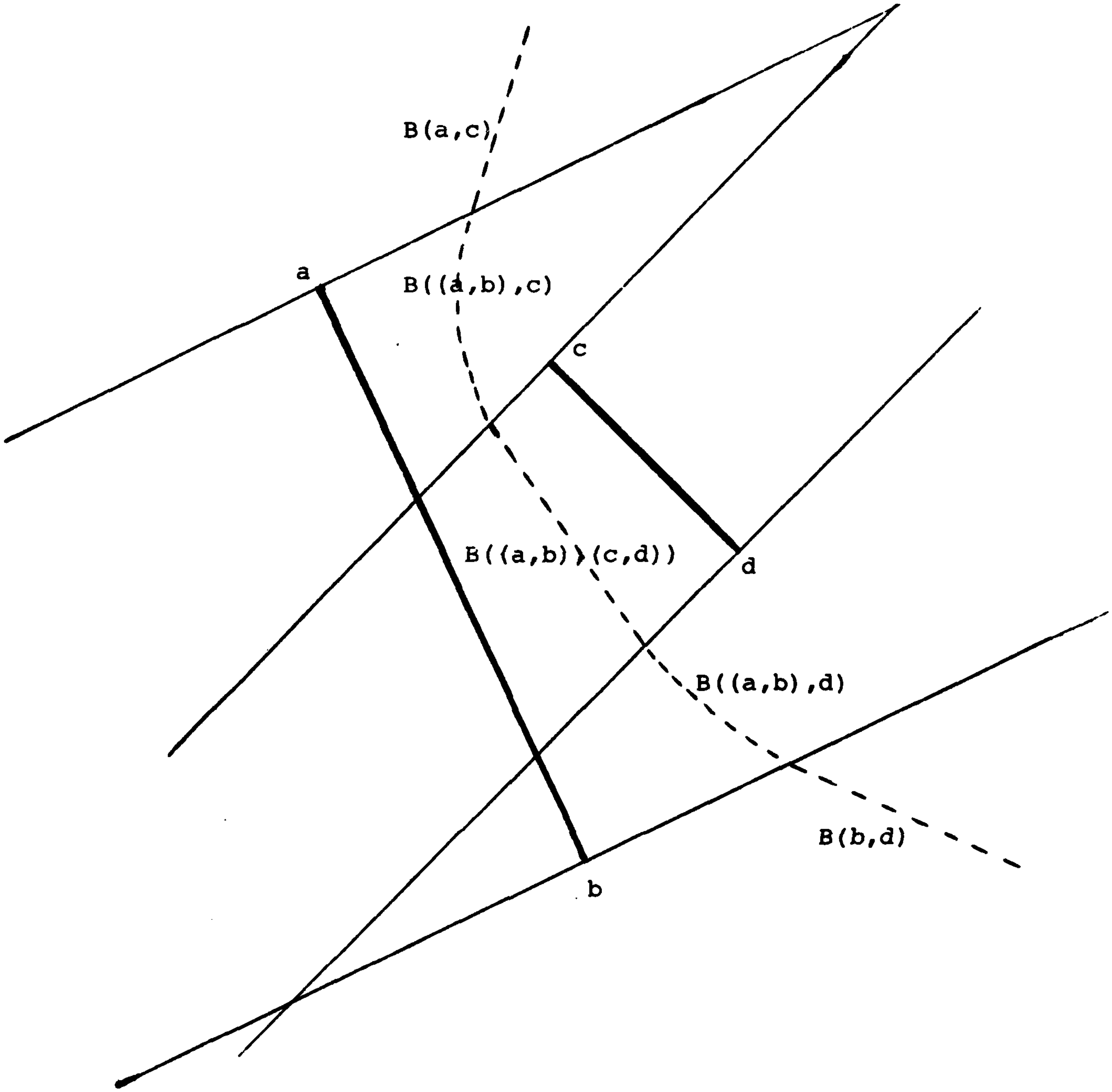
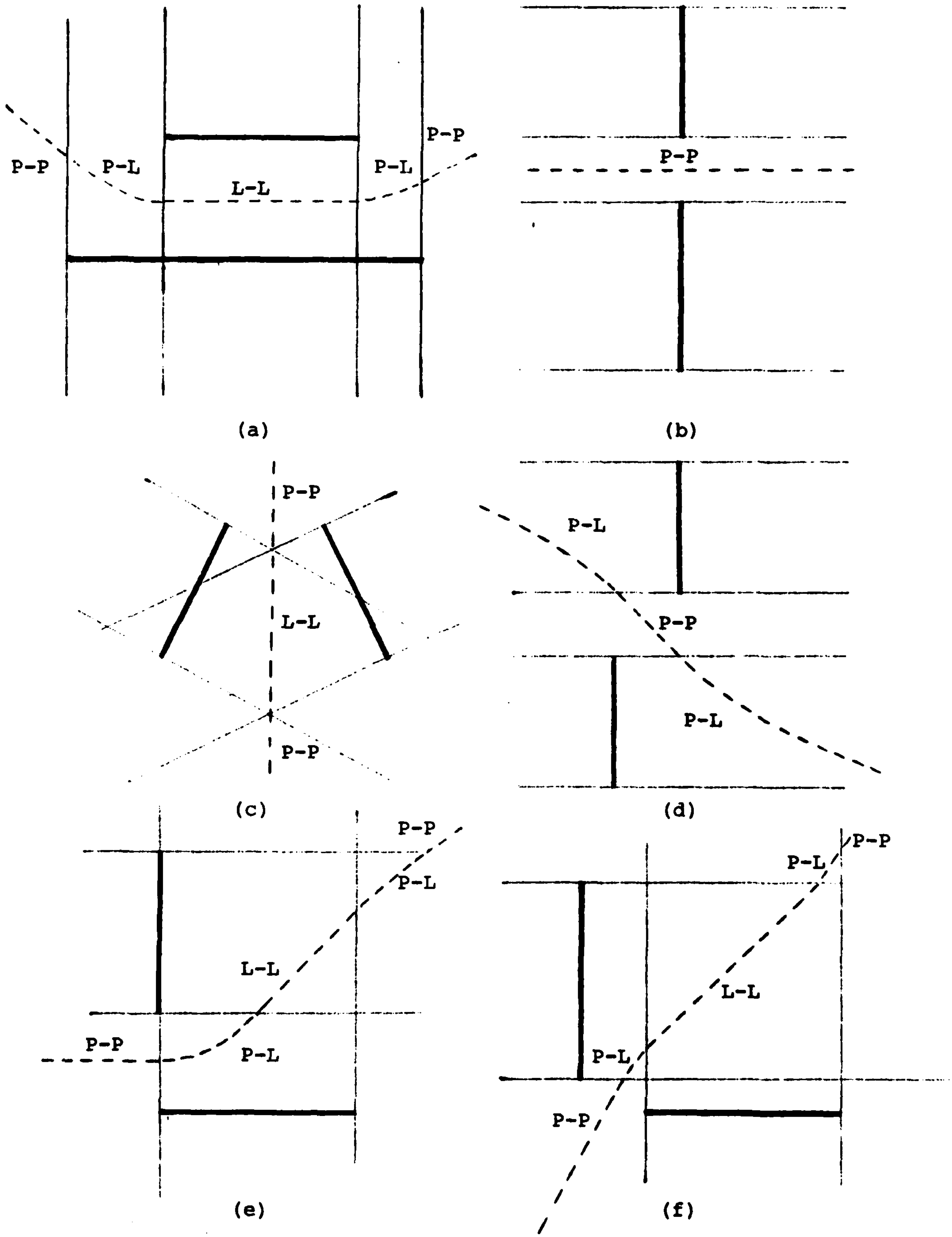


Fig. 2 Voronoi diagram of two line segments


P-P
P-L
L-L
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(a)
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(f)

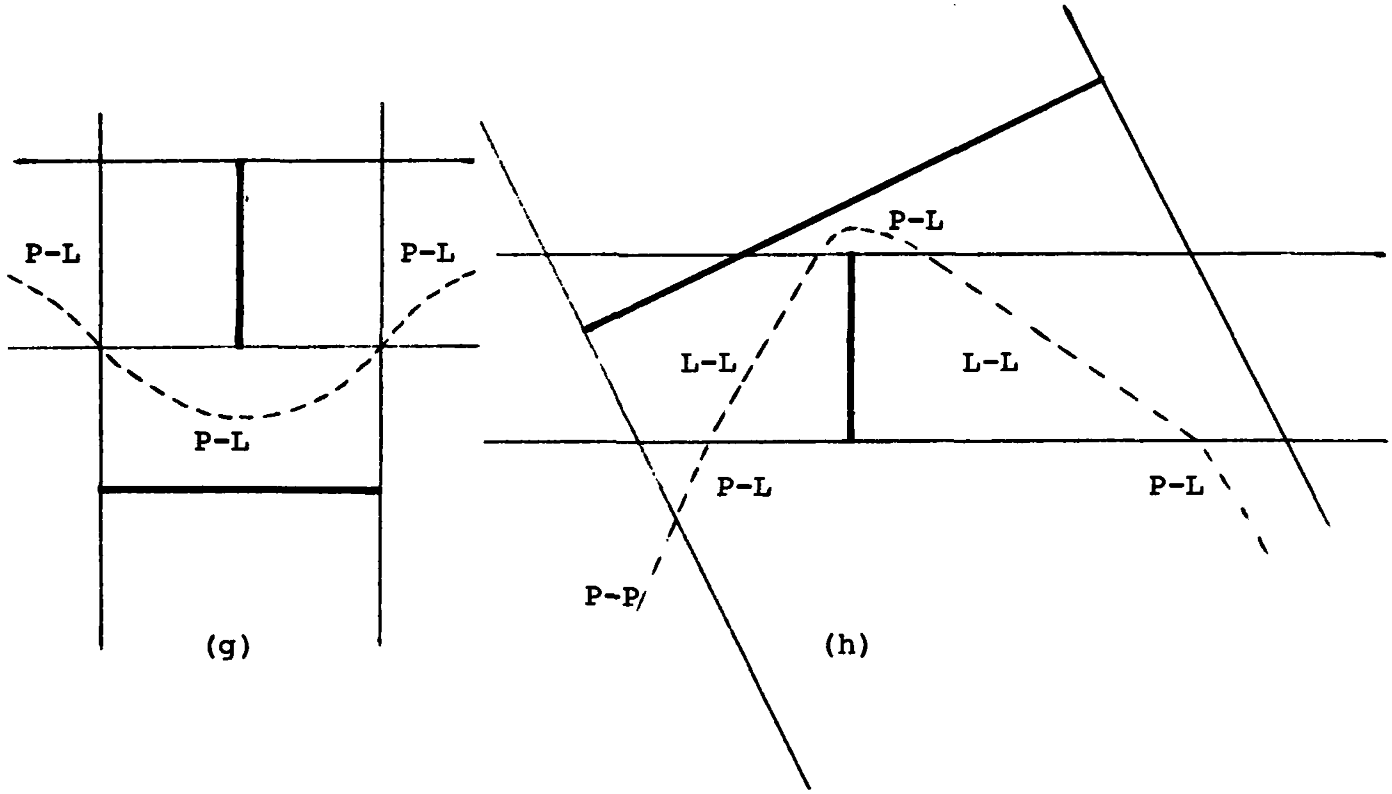


Fig. 3 Geometric relations between two line segments

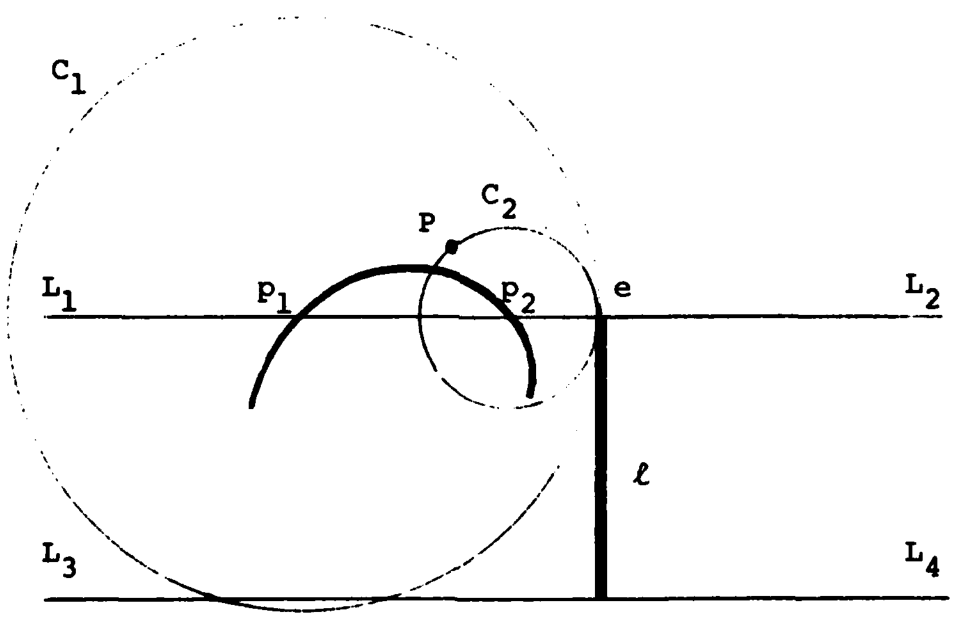


Fig. 4 Voronoi region of line segment

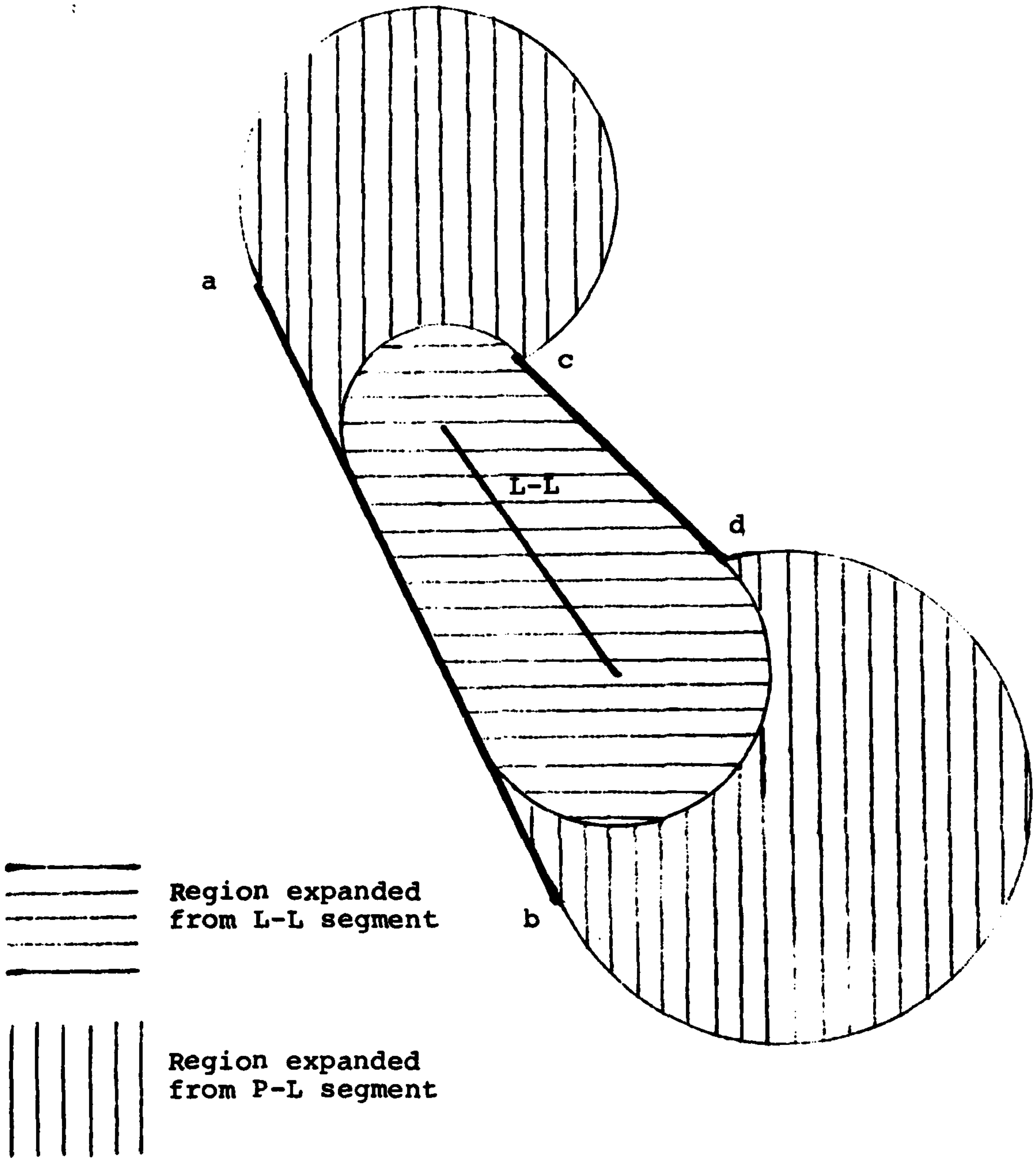


Fig. 5 Region expanded from L-L and P-L segments

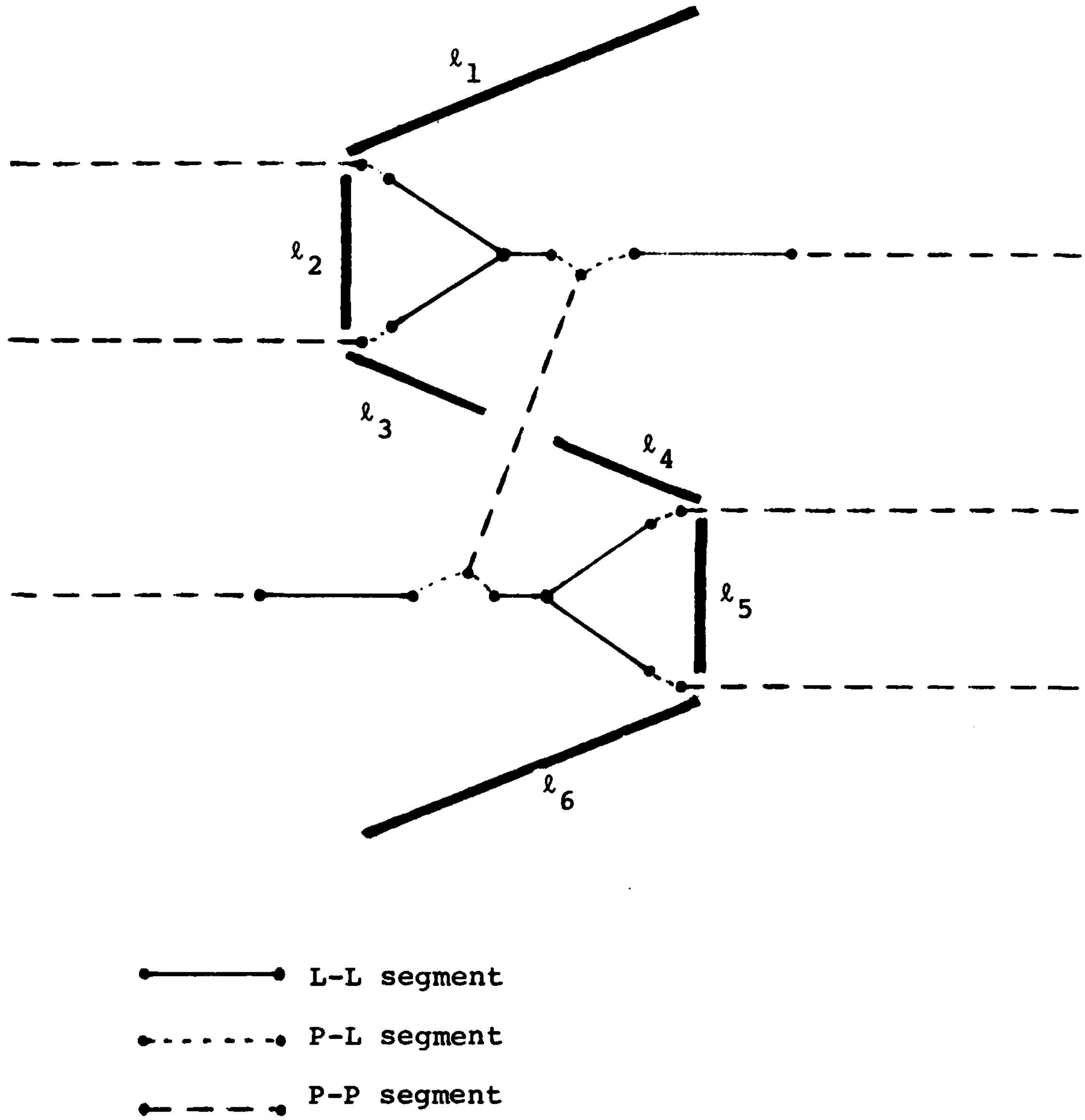


Fig. 6 Example of a labeled Voronoi diagram

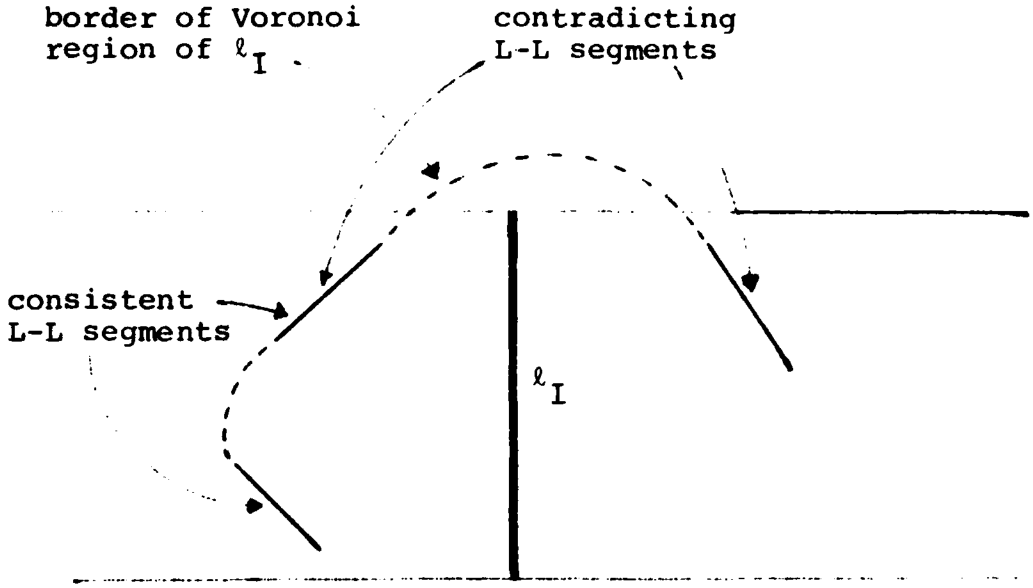


Fig. 7 Consistent and contradicting L-L segments

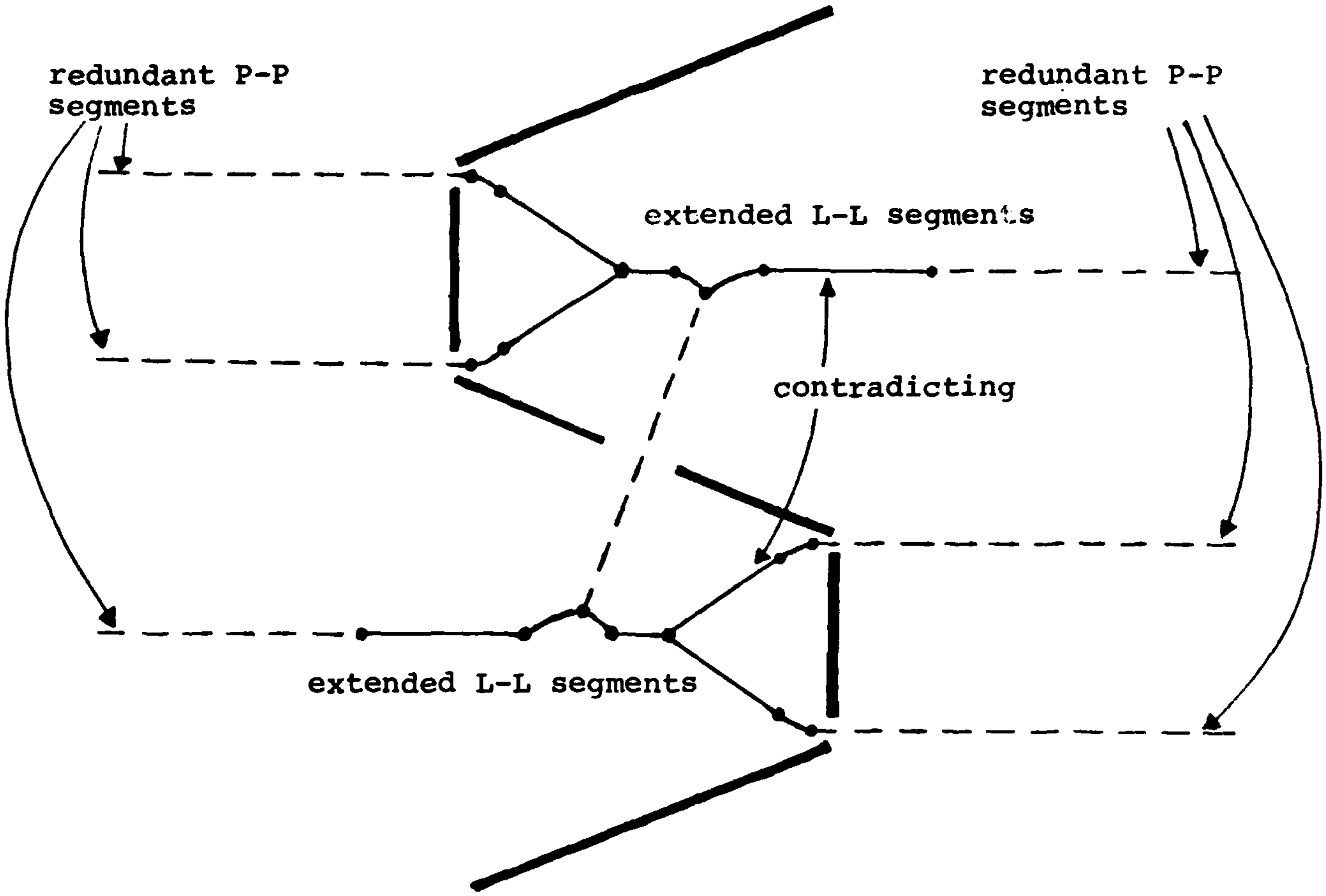


Fig. 8 Result of STEP 1-STEP 3

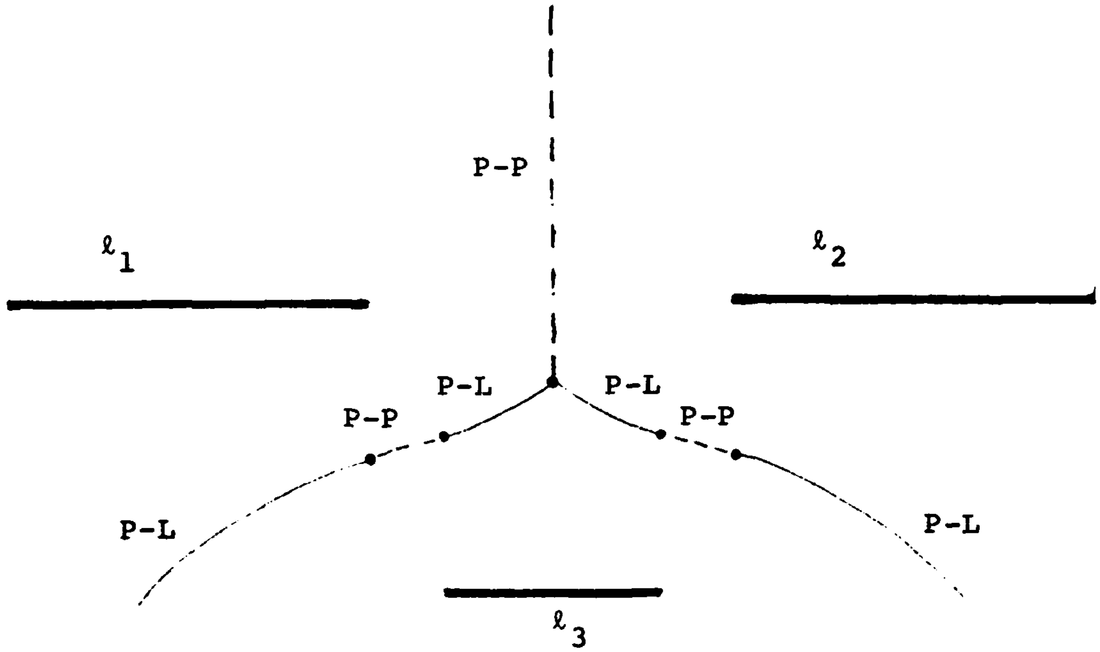


Fig. 9 Isolated P-L segments

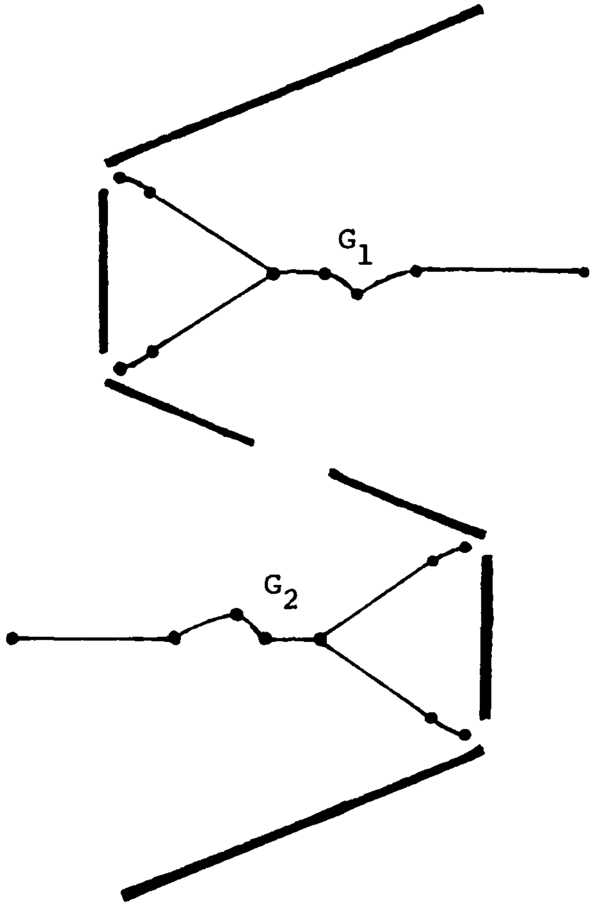


Fig. 10 Removing P-P segment connecting contradicting subgraphs

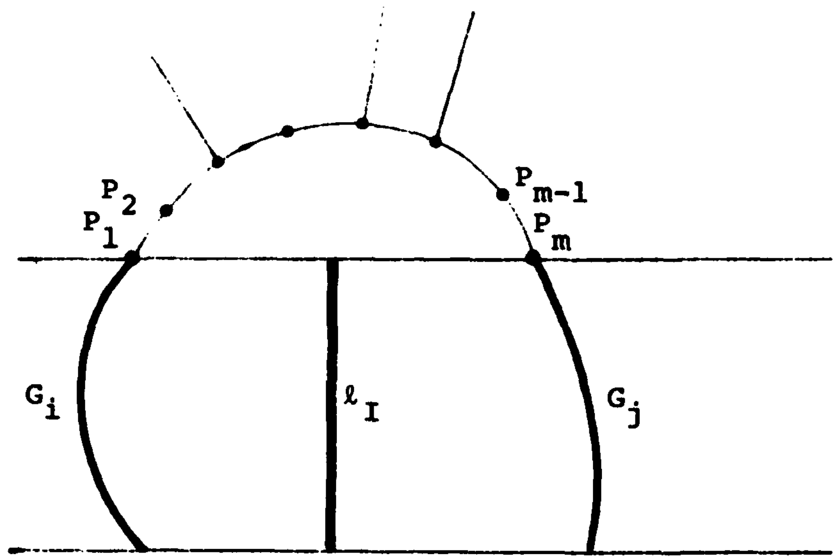


Fig. 11 Path between contradicting subgraphs

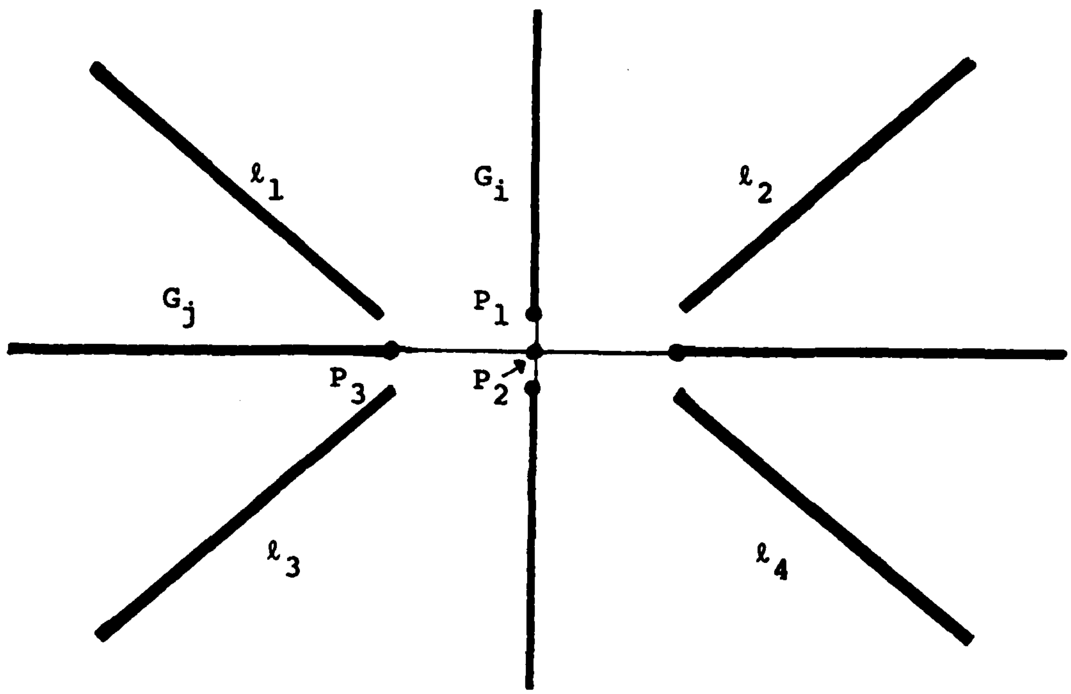


Fig. 12 Removing P-P segment

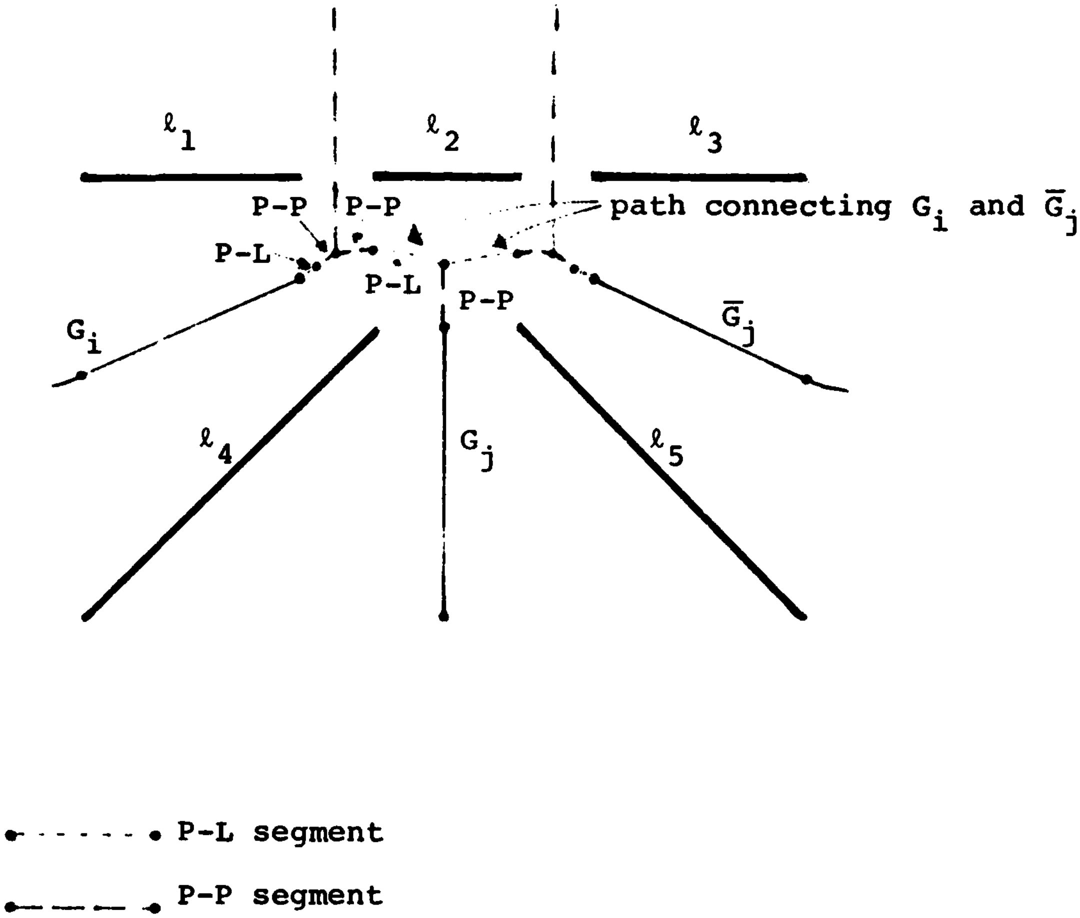
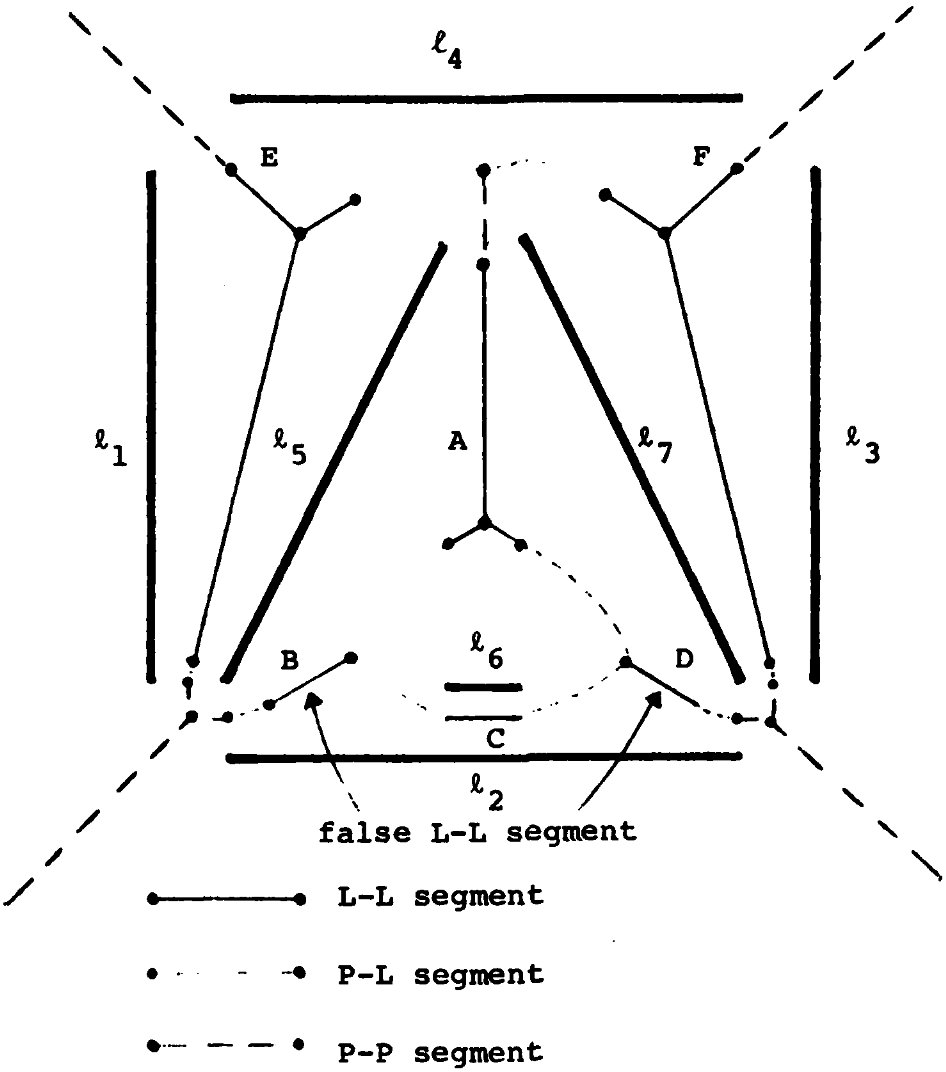
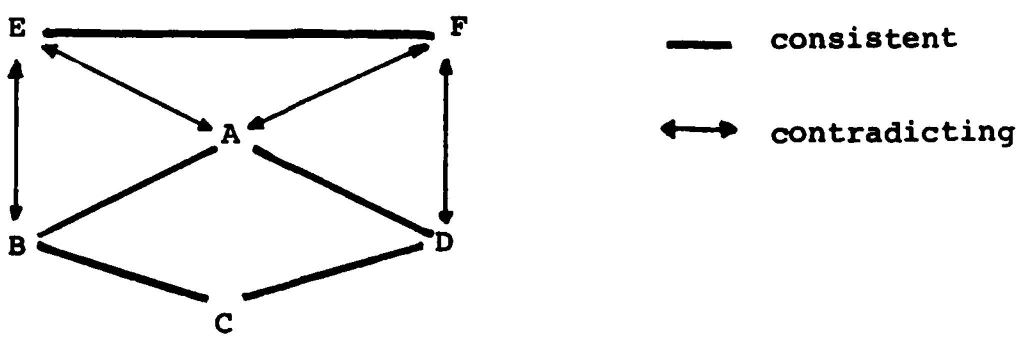


Fig. 13 Disconnecting a path connecting contradicting subgraphs

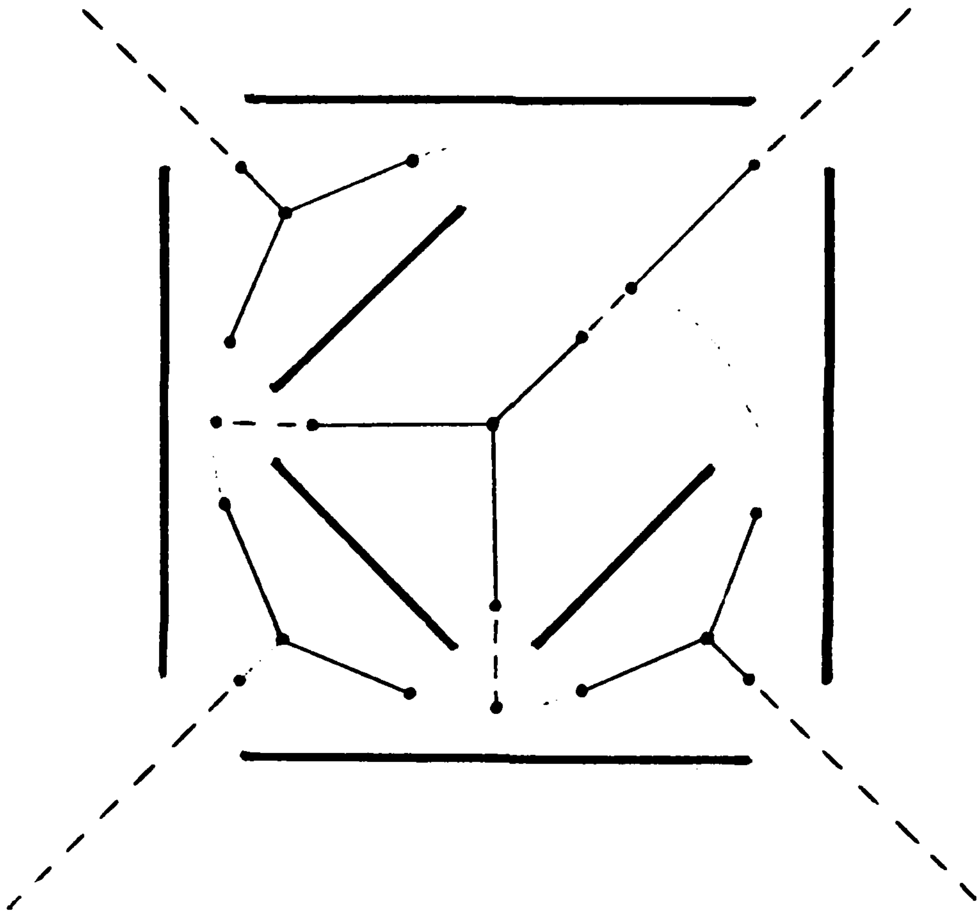


(a)



(b) relations among subgraphs

Fig. 14 A Voronoi diagram with false L-L segments



L-L segment

P-L segment

P-P segment

Fig. 15 The Voronoi diagram of a boundary with large gap: one side of the interior square is missing. Since no false L-L segment is included, the algorithm works correctly.

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Pattern recognition,
Boundary detection,
Medial axis,

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An algorithm to recover closed boundaries from disconnected boundary segments is presented. There is a close relation between the medial axis transform and the Voronoi diagram. Here we introduce a geometric labeling scheme for the Voronoi diagram of boundary segments, and recover the medial axis of closed boundaries by using the labeled Voronoi diagram. Although all examples given in this paper are pictures of straight line segments in the two-dimen-

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sional Euclidean plane, the basic idea is immediately applicable to digital pictures with curved segments. Keywords: (70 1473A)

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